Chap 2 - The Budget Constraint

→ The Budget constraint (BC) tells us how much the consumer can afford.

e.g. if walk into pizza parlor w/ $10 beers are $2, slices are $1, we could write the BC as:

$$\$2 \times \text{beer} + \$1 \times \text{pizza} = \$10$$

→ our consumption bundle is the combination of consumption goods chosen

e.g. consumption bundle is (#beers, #slices)

→ we can write the BC w/ any number of goods, but we'll usually focus on the case of 2 goods

→ this helps us so that we can represent the problem in 2 dimensions

→ 2 goods are usually enough b/c we can think about one of the goods as a composite good

→ e.g. we consume beer and everything else - everything else is a composite good

→ when we have a composite good, we usually give it a price of $1 → so it's dollars spent on everything else

→ In general, w/ 2 goods, x_1 and x_2, we'll write the BC as:

$$p_1 x_1 + p_2 x_2 \leq m$$

x_1 = quantity of good 1

x_2 = quantity of good 2

p_1 = price of good 1

p_2 = price of good 2

m = income

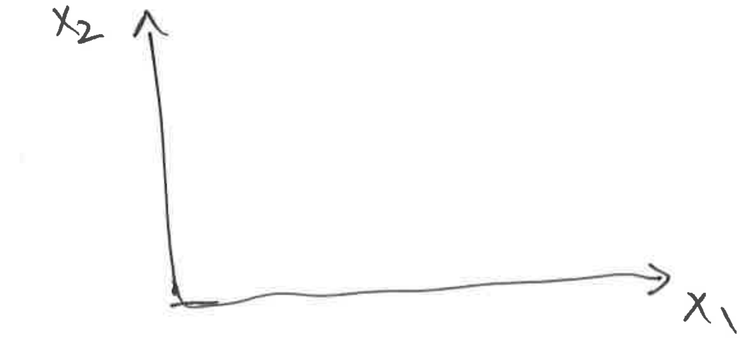
→ this equation says that the total amount spent on good 1 ($p_1 x_1$) plus the total amount spent on good 2 ($p_2 x_2$) cannot exceed total income (m)

→ We call the set of affordable consumption bundles (i.e. those that cost less than m) the consumers <u>budget set</u>

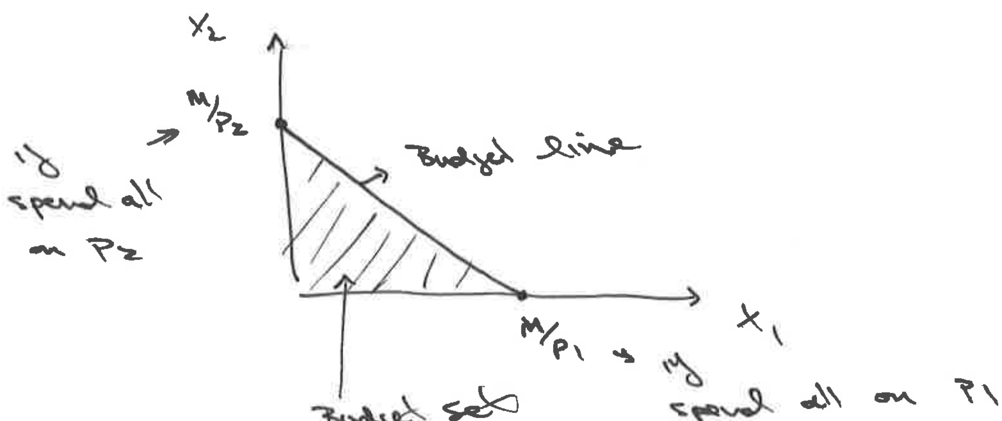
→ The consumers <u>budget line</u> is the set of bundles that cost exactly m:

$$p_1 x_1 + p_2 x_2 = m$$

Graphically, we have:



just do positive quadrant since consumption typically restricted to be non-negative



Whats the slope of the budget line?

→ write BL in slope-intercept form:

$p_1x_1 + p_2x_2 = m$

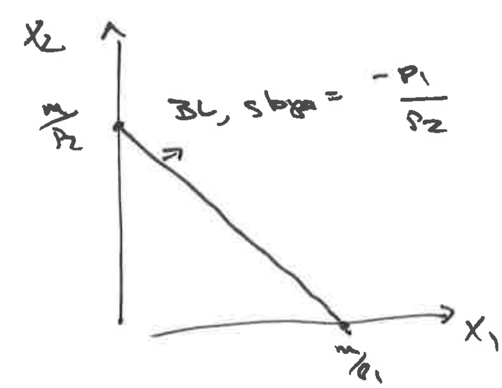
$\Rightarrow p_2x_2 = m - p_1x_1$

$\Rightarrow x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$

$y = a + bx$

$a = \frac{m}{p_2}$ = intercept

$b = \frac{-p_1}{p_2}$ = slope



Another way to find the slope of the BL:

→ consider a change in the amounts of goods 1 and 2 consumed that keeps one on the same BL:

① $p_1x_1 + p_2x_2 = m$

and

② $p_1(x_1+dx_1) + p_2(x_2+dx_2) = m$

→ dx_1 and dx_2 are changes, but they can't affect ~~total~~ spending

→ now subtract ① from ②:

$$
\begin{array}{rl}
p_1\cancel{x_1} + p_1dx_1 + p_2\cancel{x_2} + p_2dx_2 & = m \\
- \cancel{p_1}x_1 \qquad\qquad + \cancel{p_2}x_2 & = m \\
\hline
p_1dx_1 \qquad\qquad + p_2dx_2 & = 0
\end{array}
$$

$p_1dx_1 + p_2dx_2 = 0$

$\Rightarrow p_1dx_1 = -p_2dx_2$

$\Rightarrow -\dfrac{p_1}{p_2} = \dfrac{dx_2}{dx_1}$

→ $\dfrac{dx_2}{dx_1}$ gives the rate at which good 2 can be substituted for good 1 while keeping total spending unchanged

→ the slope of the BL gives the <u>opportunity</u> cost of consuming good 1 (i.e. how many units of good 2 need to be given up to consume another unit of good 1)

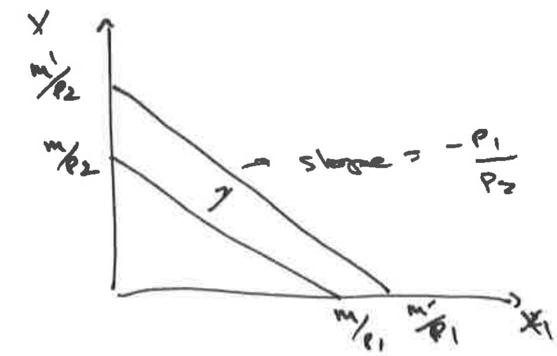
How the budget line changes

Changing income
→ ~~Shifting the~~ BL

→ If income changes, this will shift the BL in or out

→ it will retain the same slope since p_1 and p_2 didn't change

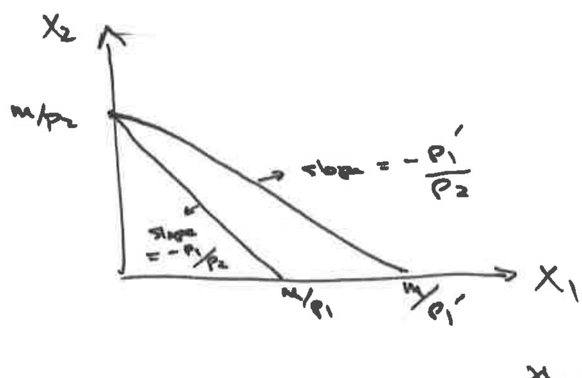
consider $m\uparrow$ from m to m':



Changing prices:
→ ~~Pivoting the BL~~.

→ if prices change, this may affect the slope of the BL

→ consider a $\downarrow$ in p_1 from p_1 to p_1':



→ in this case, the BL pivots out

→ What happens if both prices change?

→ consider case where p_1 and p_2 go up by same amount

→ say both t times as large:

initially:

$$p_1 x_1 + p_2 x_2 = m$$

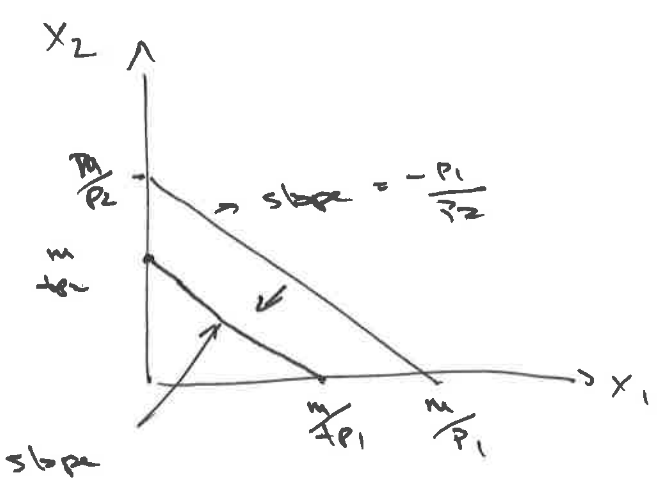
after prices change:

$$t p_1 x_1 + t p_2 x_2 = m$$

$$\Rightarrow \underbrace{p_1 x_1 + p_2 x_2 = \frac{m}{t}}_{\text{as if income became } t \text{ times smaller}}$$

⇒ this is a shift in the BL, not a pivot

→ slope still $\frac{-p_1}{p_2}$ b/c both prices changed by the same factor



slope $= \frac{-tp_1}{tp_2} = \frac{-p_1}{p_2}$

→ Summary of price changes:

→ if one price changes, the BL pivots

→ if both prices change:

→ by the same ~~amount~~ factor the BL shifts parallel to original

→ by different ~~amounts~~ factors, the BL shifts and is not parallel to the original

The Numeraire

→ Notice that if one divides both sides of the BL equation by the same factor, nothing changes.

i.e. $p_1 x_1 + p_2 x_2 = m$

$$\Rightarrow \left(\frac{p_1 x_1}{z} + \frac{p_2 x_2}{z}\right) = \left(\frac{m}{z}\right) z$$

$$p_1 x_1 + p_2 x_2 = m$$

→ Thus we can always make a normalization to our BL by setting a

1) one price equal to one:

$$p_1 x_1 + p_2 x_2 = m$$

$$\Rightarrow \frac{p_1}{p_2} x_1 + \frac{p_2}{p_2} x_2 = \frac{m}{p_2}$$

$$\Rightarrow \frac{p_1}{p_2} x_1 + x_2 = \frac{m}{p_2}$$

OR

2) by setting income equal to one:

$$p_1 x_1 + p_2 x_2 = m$$

$$\Rightarrow \frac{p_1 x_1}{m} + \frac{p_2 x_2}{m} = \frac{m}{m}$$

$$\Rightarrow \frac{p_1 x_1}{m} + \frac{p_2 x_2}{m} = 1$$

→ neither of these normalizations affect the BC at all

→ we refer to the good whose price we set equal to one as the <u>numeraire</u> good

→ why do this?
- → It is often helpful to have a numeraire good as it means there will be one less price one needs to solve for.

Representing taxes, subsidies, and rationing in a BC:

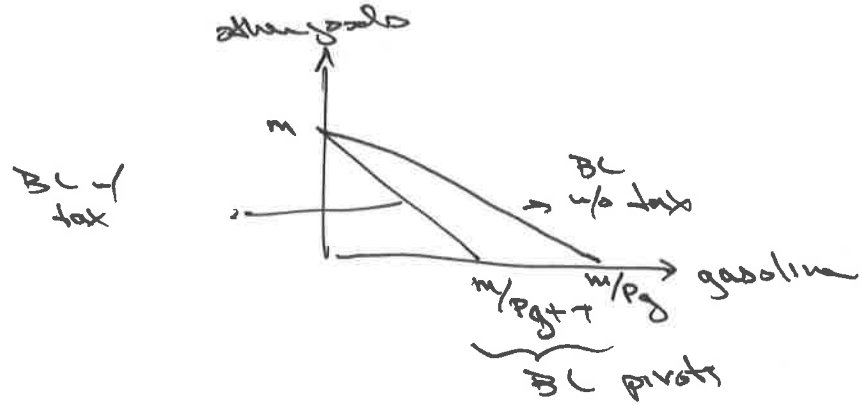
→ Taxes

→ 3 types

1) quantity tax – an amount per unit

→ after tax price = $p+\tau$, where τ = quantity tax

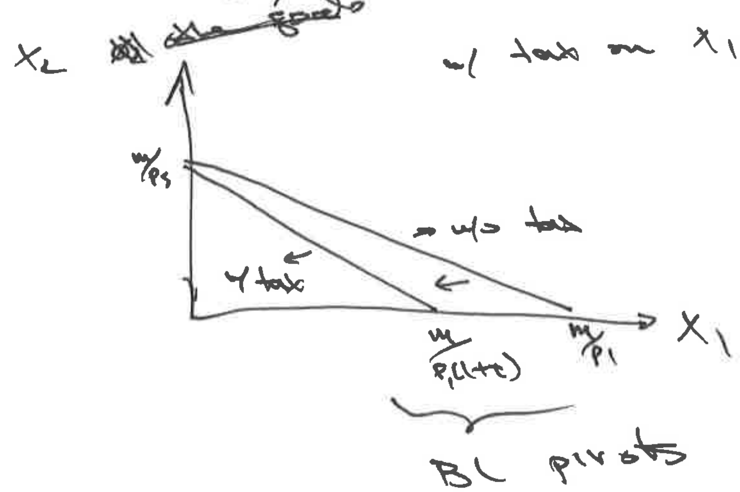
→ e.g. gasoline tax, federal = 18.4¢/gall



2) ad valorem tax – a percentage of the sale price

→ after tax price = $(1+\tau)p$, where τ = ad valorem tax rate

→ e.g. a sales tax

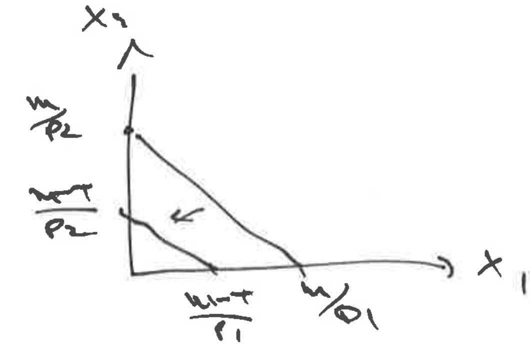


3) A lump sum tax - same amount regardless of behavior (e.g regardless of quantity purchased)

e.g. none

→ these shift the BC b/c affect income:

$p_1 + p_2 = m - T$
where T = lump sum tax amount



→ Subsidies

→ like negative taxes

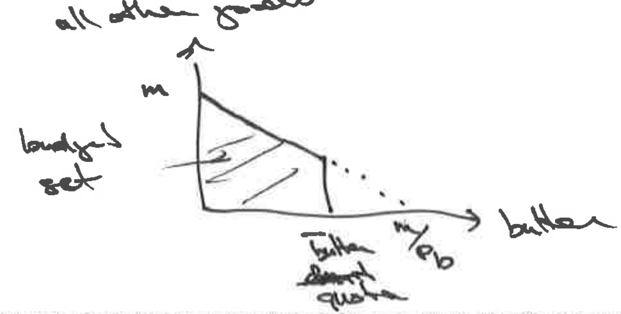
→ maybe quantity, ad valorem, lump sum

→ e.g. 1st federal tax credit on certain energy efficient appliances

→ Rationing

→ limits on quantities consumed

→ e.g. butter during WWII

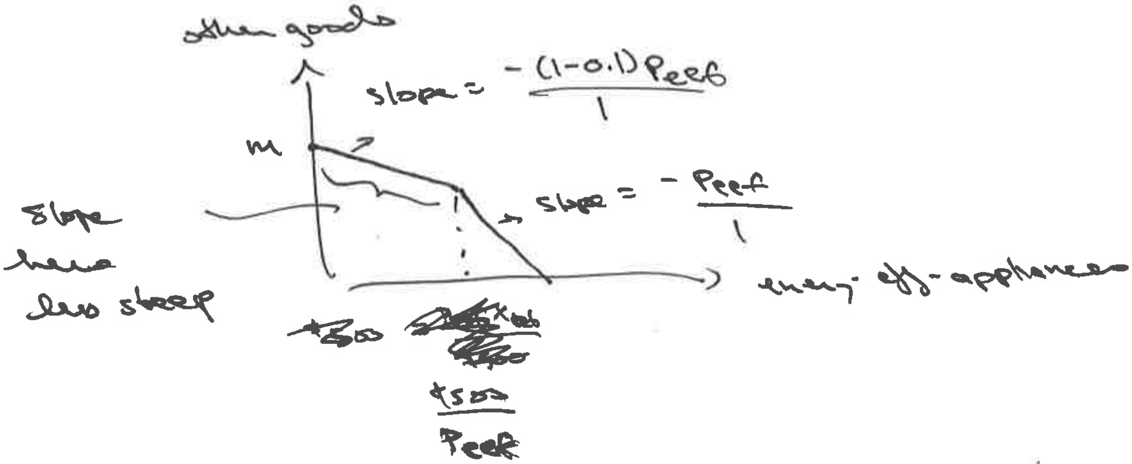


Kinked Budget constraints

→ Budget constraints may exhibit kinks if prices (before or after tax) change as quantities consumed change.

→ a kink is a point where the slope of the BL changes

→ e.g 10% tax credit on energy efficient appliances only applies for cost up to \$500



→ Kinked BL's will introduce some complications to our solution of the consumer's problem

→ we can still solve, just less straight-forward